Assignment of Unit - 1

Q-1 Evaluate $A^3 - 3A + 9I$, if I is the unit matrix of order-3 and $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$.

2. If A and B are symmetric matrices, prove that $(AB - BA)$ is a skew-symmetric matrix.

3. Prove that if A skew-symmetric, then $B'AB$ is skew-symmetric.

4. Express the following matrix as the sum of a symmetric and a skew-symmetric matrix.

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}.$$

5. Reduce the matrix A to triangular form,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$$

6. Find the inverse of the following matrices by using elementary row operations:

(i) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{bmatrix}$

7. Find the nullity of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 9 \\ 4 & 5 & 7 & 6 \end{bmatrix}_{2\times 4}$$

(8) Using elementary transformation, reduce the following matrices to the canonical form:

(i) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

(9) Find non-singular matrices P and Q such that PAQ is in the normal form for the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

and also find A^{-1}.

(10) Find the rank of the following matrices by using elementary row operations.

(i) $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

(11) Using matrix method, show that the equations

$$3x + 3y + 2z = 1, \quad x + 2y = 4, \quad 10y + 3z = -2$$
$$2x - 3y - z = 5$$

are consistent and hence obtain the solution for x, y and z.

(12) Test the consistency of the following system of linear equations

(i)
$$x + y + z = -3$$
$$3x + y - 2z = -2$$
$$2x + 4y + 7z = 7$$

(ii)
$$4x_1 - x_2 = 12$$
$$-x_1 + 5x_2 - 2x_3 = 0$$
$$-2x_2 + 4x_3 = -8$$

(13) Determine the values of λ and μ such that the system

$$2x - 5y + 2z = 8$$
$$2x + 4y + 6z = 5$$
$$x + 2y + \lambda z = \mu$$

has (i) no solution (ii) a unique sol^n.

(iii) infinite no. of sol^n.

(14) (i) Find the value of k so that the equations

$x + y + 3z = 0$, $4x + 3y + kz = 0$, $2x + y + 2z = 0$

have a non-trivial sol^n.

(ii) Determin 'b' such that the system of homogeneous equations

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + bz = 0$$

has (a) trivial sol^n. (b) non-trivial sol^n.

Find the non-trivial solutions using matrix method.

(15) Find whether or not the following set of vectors are linearly dependent or independent.

(i) $x_1 = (1,1,1,1)$, $x_2 = (0,0,1,1)$, $x_3 = (0,0,0,1)$

(ii) $x_1 = (1,1,0)$, $x_2 = (1,0,1)$, $x_3 = (0,1,1)$

(16) Find the value of λ for which the vectors $x_1 = (0,1,a)$, $x_2 = (1,a,1)$ and $(a,1,0)$ are linearly dependent.

(17) (i) If A is Hermitian matrix, then show that iA is skew-Hermitian

(ii) show that $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is skew-Hermitian and also unitary.

(18) Find the eigen-value and eigen-vector of the matrix

(i) $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 2 & 5 & 7 \\ 5 & 3 & 1 \\ 7 & 0 & 2 \end{bmatrix}$ (iii) $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(19) Find the eigen-values of $3A^3 + 5A^2 - 6A + 2I$

where $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

(20) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, find A^{-1} and A^4 using Caley-Hamilton's theorem. ~~Also show that for every~~

(21) If $A = \begin{bmatrix} -3 & -2 \\ -1 & 0 \end{bmatrix}$ then evaluate the expression

$A + 5I + 2A^{-1}$.